

## Assignment for class 12

**General direction for the candidate:** Notes provided must be copied in maths copy and then homework should be done in the same copy.

Determinant (continued): theorems and application

**Theorem 4. ( Note: proved in video link provided by school)**

If A,B,C are all square matrices of same order n and A is non- singular matrix , then

i)  $AB= AC \Rightarrow B=C$  .....(left cancellation law)

ii)  $BA=CA \Rightarrow B=C$ .....( right hand cancellation)

**Ex 4.4 Q25i)** Find matrix A Satisfying the matrix equation  $\begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} A \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\text{Let } B = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 4 & 7 \\ 3 & 5 \end{bmatrix}$$

Given  $BAC=I$

$$\Rightarrow B^{-1}(BAC) = B^{-1}I \Rightarrow (B^{-1}B)(AC) = B^{-1}I$$

$$\Rightarrow (I)(AC) = B^{-1} \Rightarrow AC = B^{-1}$$

$$\Rightarrow (AC)C^{-1} = B^{-1}C^{-1} \Rightarrow A(CC^{-1}) = B^{-1}C^{-1}$$

$$\Rightarrow A(I) = B^{-1}C^{-1}$$

$$\Rightarrow A = B^{-1}C^{-1}$$

$$\text{since } |B| = -1, \text{ adj } B = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}$$

$$\Rightarrow B^{-1} = \frac{1}{-1} \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$$

$$\text{similarly } |C| = -1, \text{ adj } C = \begin{bmatrix} 5 & -7 \\ -3 & 4 \end{bmatrix},$$

$$\Rightarrow C^{-1} = \begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$$

$$\text{Since } A = B^{-1}C^{-1}$$

$$\Rightarrow A = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} -5 & 7 \\ 3 & -4 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 21 & -29 \\ -13 & 18 \end{bmatrix} \text{Ans}$$

$$\text{Q22.i) } A^2 = AA = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}, 4A = \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix}, 5I = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

To find  $A^{-1}$  By using  $A^2 - 4A - 5I = 0$

$$\Rightarrow 5I = A^2 - 4A \Rightarrow I = \frac{A^2 - 4A}{5} = \frac{A(A - 4I)}{5}$$

$$\Rightarrow A^{-1}I = \frac{A^{-1}A(A - 4I)}{5} = \frac{(A - 4I)}{5}$$

$$\Rightarrow A^{-1} = \frac{1}{5} \left( \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix} \right)$$

$$\Rightarrow A^{-1} = \frac{1}{5} \begin{bmatrix} -3 & 2 & 2 \\ 2 & -3 & 2 \\ 2 & 2 & -3 \end{bmatrix} \text{Ans}$$

$$\text{Q8iii) } A^4 = I \Rightarrow A^{-1}I = A^{-1}A^4 \Rightarrow A^{-1} = (A^{-1}A) A^3 \Rightarrow A^{-1} = I A^3$$

$$\Rightarrow A^{-1} = A^3 \text{ Proved}$$

**Homework: Ex 4.4 8ii) Q24.ii) Q25. ii) Q21.ii), Q22.ii) Q23.iii)**

**Theorem 5. ( Note: proved in video link provided by school)**

**If A, B are inversible square matrix of same order, then AB is also inversible and  $(AB)^{-1} = B^{-1}A^{-1}$**

**HOMEWORK: Ex 4.4, Q20.**

**Theorem 6 :** If A is invertible matrix, then  $A^T$  is also invertible, and  $(A^T)^{-1} = (A^{-1})^T$

( Note: proved in video link provided by school)

**Theorem 7** If A is non singular matrix of order n, then  $|\text{adj } A| = |A|^{n-1}$  **Note : proved in video link provided by school)**

**For Example** If A is non singular matrix of order 3, then  $|\text{adj } A| = |A|^2$

**Q5. ii)** Given  $|A| = -5$ , A is square matrix of order 3

$$|\text{adj } A| = |A|^2 = (-5)^2 = 25$$

**Q6.ii)** Without computing  $\text{adj } A$ , find  $|\text{adj } A|$  If  $A = \begin{bmatrix} -2 & 0 & 0 \\ 3 & 4 & 0 \\ 10 & -7 & 3 \end{bmatrix}$

We know  $|\text{adj } A| = |A|^{n-1}$  Since order of matrix is (n) = 3

$$\Rightarrow |\text{adj } A| = |A|^2 \Rightarrow |A| = (-2)(12) = -24$$

$$\Rightarrow |\text{adj } A| = (-24)^2 = 576 \text{ Ans}$$

**Homework : Q5.iv), Q6.iii)**